Low Momentum NN-interactions and Nuclear Matter calculations.

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Separable nucleon-nucleon potentials are calculated using inverse scattering techniques as presented in previously published work. The dependence of the potentials on the momentum cut-off of the scattering phase-shifts is studied. Some comparison is made with the V_{low-k} potential. The effect of the cut-off on nuclear matter binding energy calculated by standard Brueckner theory is also presented. It is found that a cut-off larger than about 4 fm^{-1} will keep the error to within one MeV around saturation density. While the potentials are cut-off dependent the effective interaction represented by the Brueckner K-matrix is less sensitive to this cut-off. This is in particular found to be the case for the $^1S_0$ state.

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I. INTRODUCTION

EFT methods promise to provide information on NN-interactions for studies of low energy nuclear systems. Traditional methods used to obtain NN-potentials consist of a mixture of contributions from meson-exchange forces and phenomenological adjustments. The OPEP part of the potential is fairly well established while the main uncertainty concerns the short-ranged repulsion. The philosophy behind the EFT approach is that low-energy nuclear phenomena should be independent of the details of the short-ranged repulsions which are of a high energy origin and can be integrated out. This is somewhat akin to the features of the Moszkowski-Scott separation method [1] where it is explicitly shown that although the short-ranged correlations associated with the short-ranged repulsions are to a large degree responsible for the saturation of nuclear matter. The details of the repulsions are however not important, but are absorbed by an integrated quantity referred to as the "wound-integral". It provides for a density-dependent repulsion in the in-medium effective interaction to be supplemented with the long-ranged part that the method prescribes.

Related to the above one may ask about the relation between nuclear matter binding/saturation and the NN-repulsions as exhibited by the scattering phase-shifts of high momenta. Some scattered information is available from the more than 50 years of publications on nuclear matter calculations. It seems however appropriate to (re)examine this relationship by studying the effect of nuclear matter binding/staturation on the momentum cutoff of the scattering phaseshifts entering the construction of the NN-potentials.

Because the S states have poles near $E = 0$ a reasonable ansatz is to assume that these potentials are separable. [2] In a previous publication [3] a separable potential for all states considered (21) was calculated using established inverse scattering techniques. The input was experimental scattering phase-shifts and deuteron data. This does of course not provide a unique solution. One important aspect of this work is however that the phase-shifts are fitted EXACTLY. The Arndt set of phase-shifts was chosen [4]. These provide on-shell data while off-shell parts of the interactions are constrained essentially by the chosen rank of the potentials. The rank of the potentials in the different states was kept as low as was possible while complying with the experimental input. By increasing the rank the off-diagonal parts can be modified as found suitable, but not much experimental information is available here. As already pointed out by Tabakin [5] the off-diagonal parts are also affected by the chosen momentum (energy) cut-off. The binding energy of nuclear matter was calculated by standard Brueckner techniques. Comparison with calculations using the Bonn-potential [6] showed excellent agreement except in the $^3P_1$ state. In this state the half-shell reactance matrix elements showed a large difference from those of ref. [6].

This method of constructing the NN-potential is very suitable for the present study of momentum cut-offs, because of the ease of the calculations while maintaing exact fits to experimental data. The Arndt phases are defined up to 1.6 GeV lab energy which converts to $k \approx 4.3 fm^{-1}$ in the C.M. system. In the previous work the difference between some extrapolations up to $10 fm^{-1}$ was investigated. It was found although that the resulting potentials changed appreciably there was little or no change in the corresponding (half-shell) reactance matrices for momenta relevant to low energy nuclear physics such as nuclear saturation.

This finding is explored here in some more detail showing nuclear matter bindings and saturation curves for cutoffs as low as at $2 fm^{-1}$. The corresponding potential parameters are also shown as a function of cut-offs and compared with V_{low-k}.

II. NUMERICAL RESULTS

All calculations were done essentially as in the previous work. [3] It may be of interest that the time to compute the separable potentials from the 21 phaseshifts AND a many-body Brueckner self-consistent calculation at a fixed density only takes seconds which is one reason why an investigation of this type is feasible.

A. The $^1S_0$ potential and K-matrix

The separable potential is calculated for different values of the cut-off momentum of the phase-shifts. The potential is consequently only defined for momenta smaller than (or equal to) the chosen cut-off. If the cutoff-momentum given in the C.M. system is larger than the maximum momentum $4.3 fm^{-1}$ of the Arndt phases they are extrapolated by a straight line to the chosen cut-off momentum. Because one incentive for this investigation has been the development of the low momentum NN-interaction referred to as $V_{low\ k}$ [7–9] Fig 1 shows $V(k=0,k=0)$ as a function of the cutoff Λ calculated by inverse scattering from the $^1S_0$ phase-shifts.

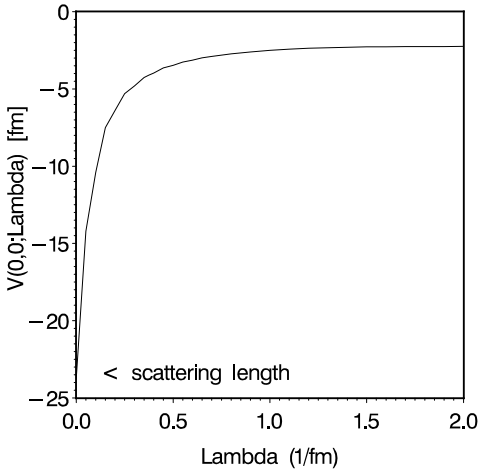

FIG. 1. $V(k=0,k=0)$ as a function of the cutoff Λ.

One finds that it is practically identical to the result shown in published papers on $V_{low\ k}$. At the limit $\Lambda \Rightarrow 0$ one finds $V(0,0) = -23.73$ i.e. the scattering length.

A plot of the diagonal elements of the same separable potential $V(k,k)$ with $\Lambda = 2.$ is shown by Fig 2. This also shows almost perfect agreement with the cited publications. It is probably not necessary to point out the completely different method used here as compared to that of the $V_{low\ k}$ derivation. The only similarity is really that both rely on fit to experimental scattering data; which in our case is exact.

The above results serve as a comparison with $V_{low\ k}$. However the main purpose here is to investigate in more detail the relation between cut-offs Λ and the potential as well as the in-medium interaction (The Brueckner K-matrix) and nuclear binding.

Fig 3 shows the dependence of the diagonal parts of the $^1S_0$ potential. One finds a rather strong dependence.

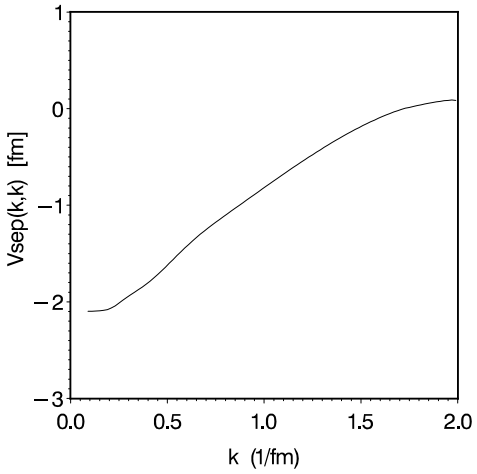

FIG. 2. The separable potential $V(k,k)$ in the $^1S_0$ state for a cut-off $\Lambda = 2.$.

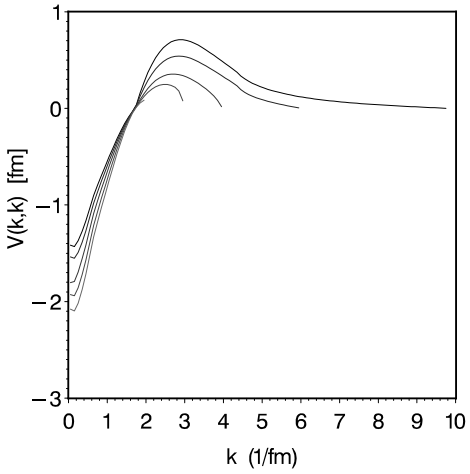

FIG. 3. The diagonal elements of the separable $^1S_0$ potential as a function of momentum k for different values of cut-off Λ. Each curve ends at its value of Λ.

The dependence on cut-off is even more striking in ccordinate space as shown in Fig 4. Shown here is $\pm v(r)^2$ where $v(r)$ is the fourier transform of $v(k)$ defined in [3]. The sign is chosen to be the sign of $v(r)$.

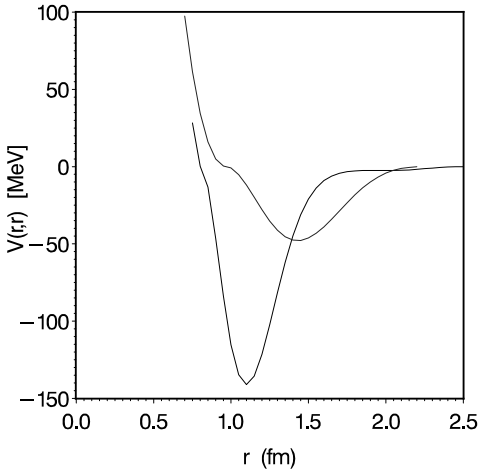

FIG. 4. Separable $^1S_0$ potential in coordinate space for $\Lambda = 6.$ (lower curve) and for $\Lambda = 4.$ (upper curve).

Quite a different Λ-dependence is shown by the Brueckner K-matrix

$$K = V + V\frac{Q}{e}K \tag{1}$$

that is used to calculate the nuclear matter binding energy. Fig 5 shows the diagonal element $K(\omega = -100MeV, k, k)$. One finds it to be practically independent of the cut-off. This can to some extent be explained by the close relationship between the K- and the R-matrix, the latter being defined by

$$R = V + VP\frac{1}{e_0}R \tag{2}$$

Here P denotes principal value which in general is not necessary to include in the definition of the K-matrix. The main difference between the two is in the Pauli-operator Q and the "e" in the Brueckner reaction-matrix that includes a (selfconsistent) mean field while e_0 in the R-matrix only has kinetic energies. It is important to know that the diagonal elements $< k|R|k > \propto tan(\delta(k))/k$ and therefore independent of the potential within the range of momenta for which the phases are fitted. For nuclear matter (or finite nuclei) with densities below or slightly above saturation only phases below $k \leq 2fm^{-1}$ are required. As pointed out above the $^1S_0$ potential is well appproximated by a separable potentiali. Therefore the R-matrix is also separable and thus completely determined by the phase-shifts and independent of any potential-representation as long as the phases are fitted. Because of the relation

$$K = R + RP(\frac{Q}{e} - \frac{1}{e_0})K \tag{3}$$

K is also separable and independent of the potential. Because the separable aproximationis good for S-states the off-shell part of the K-matrix needed in eq (3) (or eq (1) is also determined by the phase-shifts to a good approximation.

In a previous paper eq (3) was used for all states not only for the $^1S_0$ with surprisingly good results [13]. This can also be understood from the fact that the phase-shift approximation $K \sim R \propto tan(\delta(k)/k$ is a good approximation for practically all states except for the two S and the $^3P_1$ states. This will be shown below.

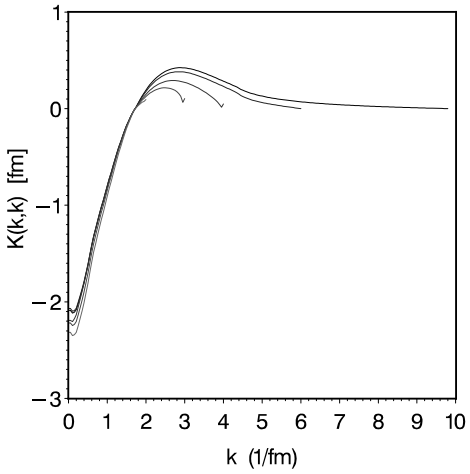

FIG. 5. The diagonal element of the Brueckner reaction-matrix in the ${}^1S_0$ with a starting energy of $-100Mev$ for different values of Λ.

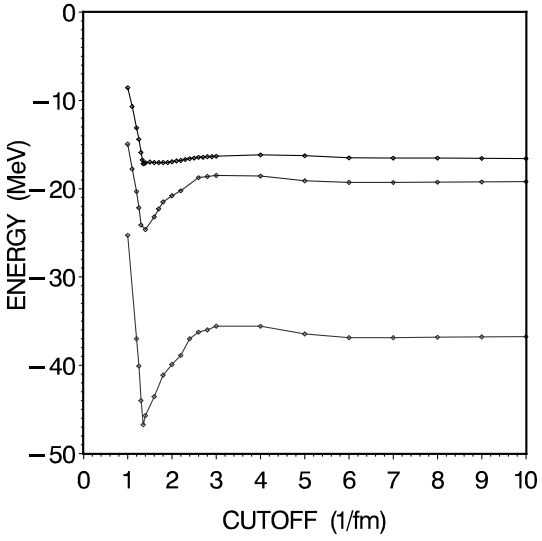

FIG. 6. The three curves show potential energies per particle as a function of cutoff Λ of the phase shifts. The upper curve is for the ${}^1S_0$ state the middle for the ${}^3S_1$ and the lowest includes all the 21 states used. This is for $k_F = 1.35fm^{-1}$.

B. In-medium Interaction and Nuclear matter

The main contributions to the binding energy of nuclear matter comes from the S-states. In Fig 6 the uppermost curve shows the contribution to the potential energy per particle from the $^1S_0$ state as a function of the momentum cut-off Λ of the phase-shifts. For momenta $k > 4.3fm^{-1}$ a straight line extrapolation to zero at the cut-off is used. The density is here fixed at a fermi-momentum $k_F = 1.35fm^{-1}$ and the mean-field is that given by the Brueckner self-consistency at the same k_F. It is seen that the energy here is almost constant to a cut-off as low as k_F. This is in agreement with Fig 5 showing the near independence of cut-off for the diagonal elements of K. The only dependence here is for low momenta which have very little weight in calculating the potentail energy. For the $^3S_1$ state (with the Bonn-B deuteron) the situation is different. A slight decrease in binding is seen already at $5fm^{-1}$ followed by a sharp increase below $2.5fm^{-1}$. The lowest curve shows the sum over all states and here the result for the $^3S_1$ is even more accentuated. Fig 7 shows the similar result for $k_F = 1.7fm^{-1}$. It is seen that the lowest momentum accceptable for cut-off has not shifted appreciably but the potential energys below $5fm^{-1}$ increase more drastically than at the lower density.

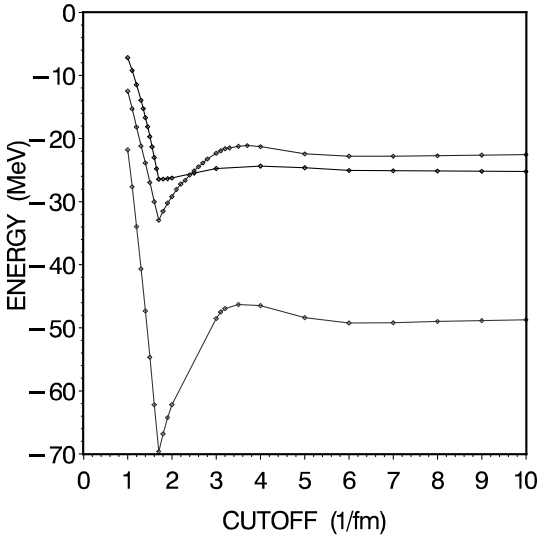

FIG. 7. Same as Fig 6 except for $k_F = 1.7fm^{-1}$

Fig 8 shows the result of binding energy calculations as a function of density. With cut-offs Λ above $5fm^{-1}$ the curves are well clustered around a common line but at a cut-off of $4fm^{-1}$ there is a noticable difference that is in accord with Figs 6 and 7. Very similar results were reported by J.Kueckei et al. [10]. It is of some interest to compare this result with a similar one where the potentials are calculated with a constant cut-off $\Lambda = 10fm^{-1}$ while in the summation over momenta when calculationg the Brueckner K-matrix the cut-off is varied. The result of such a calculation is shown in Fig 9. Here a cut-off of $4fm^{-1}$ is still acceptable but a cut-off of 3 is definitely worse and of course the $\Lambda = 2$ cutoff is very bad.

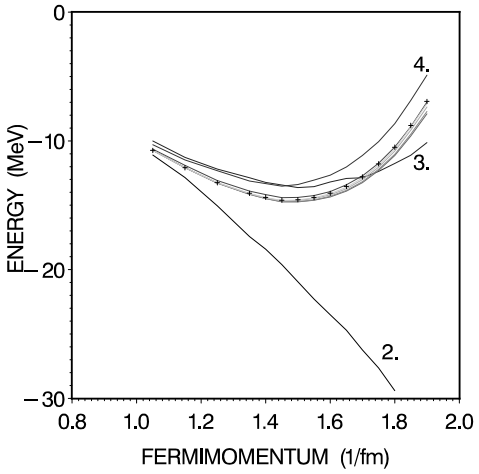

FIG. 8. The binding energy calculated by standard Brueckner method (and continuous energy spectrum) as a function of fermi-momentum. Each curve corresponds to a different cutoff Λ of the phase-shifts. For $\Lambda \geq 5. fm^{-1}$ all curves nearly coincide. The curve with crosses is for $\Lambda = 10$. For $\Lambda \leq 4. fm^{-1}$ the curves are marked.

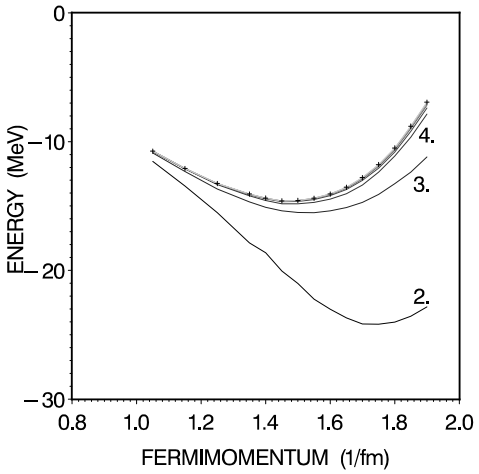

FIG. 9. Similar to Fig 8 but here all phases are cut at $\Lambda = 10. fm^{-1}$ when calculating the potentials but the cut-offs indicated in the figure is used when calculating the Brueckner K-matrix.

It was shown in an earlier work [13] that the phase-shift approximation is quite accurate for practically all states

beyond the S even at saturation density. [1] This is also seen in Fig 10 where in the center curve only the ${}^3S_1 - {}^3D_1$ the ${}^1S_0$ and the ${}^3P_1$ states are calculated in Brueckner while the remaining 18 states are in the phase-shift approximation i.e $\tan(delta)/k$. The phase-shift approximation fails badly for the three states mentioned but as shown in Fig 10 it is quite accurate for the rest. It was also found in ref [13] that a Pauli and mean field correction applied to the S-states using only the phase-shifts as input agreed surprisingly well with the standard Brueckner calculation from a NN-potential. For these states the many-body (three-body) effects due to the dispersive corrections stemming from the short-ranged correlations are very important for saturation as these corrections are proportional to the wound-integral referred to above. [1]

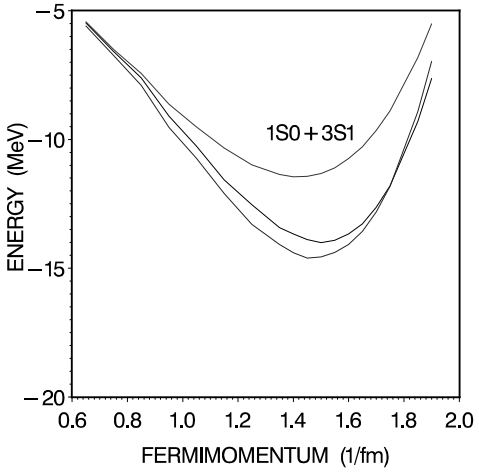

FIG. 10. The lowest curve shows the binding energy as a function of k_F with all states calculated by Brueckner while in the center curve only the ${}^3S_1$ the ${}^1S_0$ and the ${}^3P_1$ are calculated by Brueckner but all other states are by the phase-shift approximation. The uppermost curve shows the binding energy with only the ${}^1S_0$ and the ${}^3S_1$ state included.

III. CONCLUSIONS

A main goal of nuclear theory is to describe the properties of nuclei in terms of the interactions between nucleons. The nuclear many-body problem has been the subject of intense investigations and discussions starting with Brueckner's ground breaking work some 50 years ago. His work showing the importance of Pauli and dispersion effects to explain the saturation properties of nuclear matter was ground-breaking. Although much progress has been made there remains however still detailed questions relating both to the underlying interactions between nucleons and the many-body effects. With the realisation that nucleons are not elementary particles the concept of a nucleon potential becomes more obscure except (perhaps) for the long-ranged part. This will hopefully be circumvented by the EFT methods now being developped, e.g. in ref. [14]

One of the goals of the EFT methods is to develop a low momentum (long-ranged) interaction with the short-ranged part due to heavier exchange particles being integrated out. The purpose of the present investigation is to see to what extent a traditional approach based upon fits to phase-shifts depends upon the range of momenta used in the construction of a potential and subsequently the calculation of nuclear binding.

The results of this investigation as presented here relates mostly to the ${}^1S_0$ state. This may be a special case because it allows for the separable representaion to be a good approximation. It is however found in this case that

[1] Following [11,12] the approximatiom δ/k was used there.

(at least in the calculation of nuclear binding by Brueckner) the potential in itself is irrelevant. The only requirement is that it fits the phase-shifts for momenta within the region of interest which does not provide a unique solution of the potential as it depends quite strongly on the cut-off. But the separable representaion does provide a practically unique effective interaction (K-matrix) independent of cut-off.

The ${}^3S_1 - {}^3D_1$ states will be the subject of another paper. Here the biggest source of uncertainty relates to the deuteron D-state probability P_D. The results shown in Figs 6 and 7 do however indicate a much greater sensitivity to cut-offs below $4 - 5fm^{-1}$.

It was shown in Fig 10 that for higher angular momentum states (except the ${}^3P_1$) the phase-shift approximation is quite adequate.

A topic of interest is to what extent the experimentally undetermined off-shell part of the potential affects the in-medium K-matrix which because of the propagation through a mean-field is off-shell. In the results presented here this question is not answered. The off-shell K-matrix is here constrained by the choice of lowest possible rank of the interaction. It was one the incentives of this work that a change of the off-shell behavior is possible by increasing the rank of the potential while keeping the on-shell data intact. The above-mentioned question could then be answered. This would be very difficult by traditional potential construction using meson-theoretical input, but in principle easy within the separable method implemented here.

That the off-shell behavior does indeed affect the in-medium interaction is exemplified by the ${}^3P_1$ case which deviates appreciably from the Bonn-result. In the previous work with the separable potential the comparison with the Bonn-B potential half-shell reactance matrix was shown to be very different from that of the separable potential. [3] The rank-1 separable potential used in this case appears not to be appropriate.

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